Application of the Hoek & Brown (1980) failure criterion to the design of the foundation of an arch dam

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ABSTRACT: Arch dams transmit to the foundation important loads; particularly, they have a high magnitude with one determinant horizontal component. To define the optimum foundation level of these type of structures on a rock mass, it is necessary to know the orders of magnitude of the safety factors incorporating the main factors that determine the ground response: type of rock, quality of the rock media, uniaxial compression strength of the rock matrix, load inclination, depth of the foundation, inclination of the surface of contact, etc. A real case is presented showing the influence of the variations of the different parameters involved in the obtained safety factors, for the bearing capacity of the rock mass. The original Hoek & Brown (1980) failure criterion has been used together with the Serrano & Olalla (1994) methodology to calculate the ultimate bearing capacity.

1 INTRODUCTION

In all typologies of dams it is necessary to consider the dam-foundation interaction, in order to evaluate its resistance and deformability. In double curvature arch dam this is particularly important.

From the point of view of the foundation's resistance, there are two extreme situations to be avoided: ultimate shear strength (to reach the ultimate bearing capacity of the foundation as a whole) and sliding stability (both for the contact of the dam at the foundation level as well as for the discontinuities of the rock mass). In this paper, a method to calculate the ultimate bearing capacity (q_h) of a double curvature arch dam foundation is applied and the obtained safety factors are analyzed. The rock media is considered to be homogeneous and isotropic.

Beginning with the earliest geotechnical documents and up to the present, how to determine the loads that cause failure in a rock mass is very controversial and is subjected of great debate.

Several theories have been proposed, both in Spain as well as in other countries of our geographic and technological surroundings. Almost all of these theories are based on empirical considerations.

Amongst the failure laws found in literature, Hoek & Brown (1980) criterion is the most used and recommended to reproduce the response of a plastified rock mass as a whole. Otherwise, Serrano & Olalla (1994) methodology provides a theoretical framework and includes the main factors to determine, firstly the ultimate bearing capacities and secondly, the values of the safety factors obtained.

Starting of an approach to the regulations existing in Spain and in other countries, this paper determines the ultimate bearing capacity and the obtained safety factors for a foundation of a double curvature arch dam founded on sound schists.

The performed calculations allow to confirm the adopted decisions in relation to the depth of the foundation level, with the expected quality for the rock mass and its corresponding mechanics properties.

2 SIMPLE OR DOUBLE CURVATURE ARCH DAM CHARACTERISTICS

Arch dams transmit to the foundation important loads; particularly, they have a high magnitude with one determinant horizontal component.

At present, to study the stress behavior of a dam and its foundation, finite element models are usually made. After several iterations they provide the optimal shape of the dam body, which with the minimum volume reaches allowable stresses in the concrete (within the corresponding safety factors) and tolerates the stresses brought to bear on the rock mass, according to all the load hypotheses that must be taken into consideration.

The transmitted loads to the foundation for a double curvature arch dam 100 m high can be around 2-3 MPa (under the hydrostatic load hypothesis), with a 15º inclination in relation to the vertical.

Figure 1 shows an used model of finite elements for the dam-foundation interaction study.

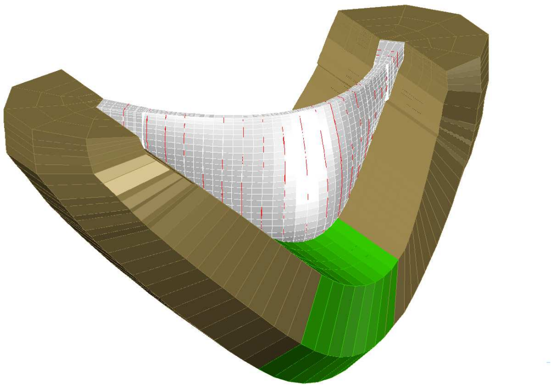

Figure 1. Example of a numerical model of a 100 m high double curvature arch dam.

3 REGULATORY FRAMEWORK

3.1 *Spanish regulations*

In Spain there are several regulatory documents used to calculate the allowable bearing capacity (q_a) and the ultimate bearing capacity (q_h = SF x q_a) of a rock media, SF being the safety factor.

When applying these different alternatives for a real dam project carried out by "Jesús Granell, Ingenieros Consultores", using the same data, significant differences arise.

The input data should be representative of the rock volume under the dam foundation, until a depth equal to 150 - 200% of the footing width.

3.1.1 *Recomendaciones Geotécnicas para Obras Marítimas. 1994. (ROM 05-94).*

Paragraph 3.5.4.2.2. titled "*Cargas admisibles en suelos cohesivos firmes y rocas*" ("Allowable loads of firm cohesive soils and rocks"), provides the allowable bearing capacity calculations as a ratio of σ_c (uniaxial compression strength of the rock matrix); for weathered rocks the suggested factor to be applied is 0.4 to 0.6 and for very fractured rocks it is 0.1 to 0.2.

It states that this percentage should be "*smaller as the degree of fracture increases and σ_c is greater*". Also states that "*in considered project situation, this procedure cannot be used when the inclination in relation to the vertical of the resultant is greater than 10% with respect to the vertical*".

Table 1 shows results obtained when this ratio ranges between 0.2 and 0.4.

3.1.2 *Guía de Cimentaciones de Obras de Carretera. 2009. (GCOC).*

Paragraph 4.5.3. "*Cimentaciones superficiales sobre roca*" ("Shallow foundations on rock") considers that rocks with σ_c > 1 MPa, RQD (Rock Quality Designation) > 10, and a degree of weathering less than IV, the allowable bearing capacity can be estimated using an expression with three factors; a_1, a_2 and a_3, respectively.

This regulation is not strictly applicable because its use is limited if the support area is less than 100 m². In spite of this limitation, the procedure of calculus is as follows:

The concept of the uniaxial tensile strength is used by means of the parameter a_1. In sedimentary rocks and in some metamorphic rocks, as the schists with vertical foliation, is proposed to assign the value of $a_1 = 0.6$. For schists with subhorizontal foliation, it is proposed a value of $a_1 = 0.8$.

The degree of weathering affects a_2 parameter. For sound rocks it assigns a value of $a_2 = 1$ and for lightly weathered rock assigns a value of $a_2 = 0.7$.

To take into account the discontinuity spacing, it is proposed the factor a_3. This value should be less between the square root of the minimum adimensional spacing and the square root of adimensional RQD index. In this case, it has been assumed that the discontinuity spacing is 0.6 m, because a minimum parameter a_3 is suggested.

To obtain the allowable bearing capacity, the applied factors have been in this case $a_1 = 0.7$; $a_2 = 0.85$; $a_3 = 0.775$, respectively.

3.1.3 *Recomendaciones Geotécnicas para Obras Marítimas. 2005. (ROM 05-05).*

In the criteria shown in paragraph 3.5.4.7. named "*Carga de hundimiento en suelos cohesivos firmes y rocas*" ("Ultimate bearing capacity of firm cohesive soils and rocks"), is observed that is very similar to GCOC proposed method. Also, the applicability of the procedure and the verification of other limit states are also very similar between both regulations.

3.1.4 *Código Técnico de la Edificación. 2006. (CTE).*

Paragraph 4.3.4., named "*Presiones verticales admisibles para cimentaciones en roca*" ("Allowable vertical pressures for rock foundations"), it is enhanced that is applicable only to rock media with σ_c > 2.5 MPa and RQD > 25%.

The simplified analytical calculation proposed is similar to the development by Canadian Geotechnical Society. It is presented in the following chapters.

A value of 0.3 is applied to uniaxial compression strength of the rock matrix to obtain the allowable bearing capacity.

Table 1 summarizes the results obtained for q_h for different values of σ_c, applied to the real case.

3.2 *International regulations*

The application of several international regulations to this particular example is summarized in the following paragraphs:

3.2.1 *Canadian Foundation Engineering Manual. 1985. Canadian Geotechnical Society. CGS.*

Paragraph 9.2 of part 2, named "*Foundation on sound rock*", is valid when the discontinuity spacing is greater than 30 cm, even if the resistance of the rock material is very weak.

It is proposed the determination of the allowable bearing capacity as a percentage of σ_c, so that with a nominal safety factor of 3, the analytical expression is function of discontinuity spacing, discontinuity aperture and foundation width.

Although a factor SF = 3 is proposed to be used, it says textually that "*the factor of safety against general bearing failure (ultimate limit states) may be up to ten times higher*".

From safe side considerations, a discontinuity spacing of 60 cm, an aperture of 0.1 mm and a 24 m wide strip footing bearing have been used. The ultimate bearing capacity obtained is shown in Table 1, for different values of σ_c. A factor of 0.3 has been applied to obtain the allowable bearing capacity.

3.2.2 *Standard Specifications for Highway Bridges. 1997. AASHTO.*

In section 4ª Foundations, in its paragraph 4.4.8.1.2 named "*Footings on broken or joined rock*", proposes again the calculation of the ultimate bearing capacity as a percentage of σ_c.

It is based on a personal communication of Hoek from year 1983, not published, where the factors to be taken into account depend on the rock type and rock quality. To quantify the rock quality three different geomechanical classifications are used, interchangeably: RMR (Rock Mass Ratting), Q (NGI) and RQD.

For type schistose rocks, according to their qualities: if is very good rock (RQD = 90-95 or RMR = 85) the proposed factor is 2.3 and if is good (RQD = 90 or RMR = 75) the proposed factor is 1.

A factor ranging from 1 to 2.3 is applied to obtain the ultimate bearing capacity.

3.2.3 *Rock Foundations. 1996. Technical Engineering and Design Guides as adapted from the US Army Corps of Engineers, Nº 16.*

Section III of chapter 6º named "*Bearing capacity*", in its paragraph C proposes that the value of the allowable load is direct and exclusive a function of the RQD value; for RQD = 90 the result for the ultimate bearing capacity is around 57 MPa. The σ_c value is not taken into consideration.

Outside other considerations, this relationship was validated for authors for rock masses with closed discontinuities or with aperture less than 1".

Table 1 presents the results obtained of q_h for different values of σ_c, applied to the real case.

3.3 *Other formulations*

The following formulations have been also analyzed:

- Bishnoi (1968)

This formulation is not applicable to this real case because the ratio between the discontinuity spacing and foundation width is less than 0.1.

- Kulhawy & Carter (1992)

The ultimate bearing capacity depends on the rock quality by GSI index (Geotechnical Strength Index), σ_c and m_i (representative parameter of Hoek & Brown criterion). The ultimate bearing capacity is obtained as a percentage of σ_c.

- Zhang & Einstein (1998)

These authors based on 39 tests obtained the following expression for ultimate bearing capacity as a function of σ_c as follows:

$$q_h = 4.8(\sigma_c)^{0.5} \qquad (1)$$

Table 1 shows the results of q_h for different values of σ_c, applied to the 100 m high double curvature arch dam case.

3.4 *Summary of results obtained for the ultimate bearing capacity*

All results obtained in previous paragraphs are shown in Table 1.

Table 1. Values obtained of ultimate bearing capacity by different regulations and formulations.

	ULTIMATE BEARING CAPACITY q_h (MPa)			
σ_c (MPa)	35	*40*	45	
RQD				*90*
ROM (1994)	21-42	*24-48*	27-54	
GCOC (2009)	8.2	*8.7*	9.9	
ROM (2005)	8.2	*8.7*	9.9	
CTE (2006)	31.5	36.0	40.5	
CGS (1985)	31.5	*36.0*	40.5	
AASHTO (1997)	35-80.5	*40-92*	45-103.5	
USACE (1996)				*57*
Kulhawy & Carter (1992)	23	*32*	65	
Zhang & Einstein (1998)	28	*30*	32	

NOTE: Allowable bearing capacity as been assumed to be $q_h/3$, with F=3. In italics type the best estimation.

4 PROPOSED METHODOLOGHY: SERRANO & OLALLA (1994, 1996 AND 2000)

4.1 *Theoretical bases*

This method is based in the application of the plasticity theory to the Hoek & Brown (1980) failure criterion. The differential equations that govern the stress field of this phenomenon are solved by the characteristic lines method.

4.2 *Assumptions*

In the same way as soils, where the calculation of ultimate bearing capacities is traditionally performed with the well known polynomial formula of Brinch–Hansen (from the plasticity theory and the Mohr–Coulomb failure criterion), in the case of rock media, it is also done using the plasticity theory together with the Hoek & Brown (1980) failure criterion.

The Hoek & Brown criterion is probably the most accepted failure criterion to reproduce the limit states of stresses in rock masses.

The original formula (1980) remains valid in this case where poorly jointed rock masses and slightly weathered are present. Subsequent improvements proposed by same authors, (mainly a different exponent of 0.5), have not practical impact in this case in the evaluation of the parameters m and s because the rock mass has good quality. Consequently, in this particular case these modifications have not importance.

Some of the coefficients that are incorporated in the classical formula of Brinch-Hansen also are taken into consideration in this method, particularly:

- Acting load inclination on foundation.
- Natural ground inclination.
- Inverted slope in foundation surface (this aspect is considered under a simplified hypothesis).
- Depth of the foundation level.

The main hypotheses are:

- Perfect plasticity theory (rigid-plastic).
- Method of calculation based on the characteristic lines theory (Sokolovski, 1965).
- Hoek & Brown rock failure criterion (1980).
- Two-dimensionality (plane strain).
- The effect of the selfweight terrain is introduced under a simplified assumption.

Consequently, the ultimate bearing capacity (q_h) that causes the plastification of the media, without selfweight contribution, has the following expression:

$$q_h = \beta(N_\beta - \zeta) \qquad (2)$$

where

$$\beta = \frac{m\sigma_c}{8}; \qquad \zeta = \frac{8s}{m^2} \qquad (3); (4)$$

Being m and s the representative parameters of Hoek & Brown (1980) criterion and σ_c the uniaxial compression strength of the rock matrix.

Load factors N_β can be obtained by means of abacus or in a more precise way by mathematical expressions, indistinctly (Serrano & Olalla, 1994).

4.3 *Incorporation of rock masses weight*

Formula (2) is completed by expression (5) where the second summand includes the effect of the self-weight of the ground.

$$q_h = \beta(N_\beta - \zeta) + \frac{\gamma B N_\gamma}{2} \qquad (5)$$

Where B = foundation width; γ = specific weight of rock mass; N_γ = classic load factor.

This load factor, for the Mohr-Coulomb failure criterion, is an exclusive function of the friction angle of the involved material. As the friction angle corresponding to Hoek & Brown hypotheses is not an unique value, because it depends on the acting stress, it is necessary to make additional assumptions. It can be incorporated in a simplified manner.

Serrano & Olalla (2002) demonstrate that assuming an angle that corresponds to the harmonic mean of the extreme values of the sines of the instantaneous friction angles, defined by the loads acting in the foundation, represents the use of the secant slope, (of the strength curve of Hoek & Brown in the defined tensional range), to the failure criterion between these points.

5 APPLICATION TO A DOUBLE ARCH DAM

The calculation of the ultimate bearing capacity of the foundation of a double curvature arch dam 100 m high has been done applying Serrano & Olalla (1994) methodology. A parametric study has been elaborated using different theoretical assumptions and different hypotheses corresponding both to geometric and to geomechanical properties of foundation.

5.1 *Main data*

As with the classical theories on Soil Mechanics, for calculating the ultimate bearing capacity with the Mohr-Coulomb failure criterion, there are many additional factors involved in the determination of the ultimate bearing capacity on rock mass.

The best estimate of the average value of each parameter involved is identified (as proposed by Serrano & Olalla (1996)), as well as some range of values that could be also interesting to study its influence on the final results.

5.1.1 *Geotechnical parameters*

These values should represent the zone mainly affected by the stresses transferred by the dam; that is, the zone corresponding to the known “pressures bulb”. Based on the elasticity theory applied for a strip load (valid for the dam foundation) is usually assumed that reaches a depth from 1.5 to 2 times the foundation width, approximately.

The involved parameters are:

a) Rock type: the corresponding parameter is m_i. In this case, the rock of foundation is identified as a metasedimentary type (schist). A value of $m_i = 12 \pm 3$ is assigned following the recommendation given by Hoek (www.rocscience.com).

Given that greater is the magnitude of this parameter greater the result obtained, in the absence of an extensive and specific triaxial tests campaign, is considered valid to adopt directly the mean value assessed to schists; $m_i = 12$.

b) Rock quality: originally, this concept was identified with RMR value. Nowadays, the Geomechanical Strength Index GSI is used for this purpose, Hoek (1994).

For the rock contact in higher sections of the dam, a value of 65 is assigned. It is considered that represents the best estimation of its mean value. Probably, the value of the so called "pressures bulb" zone is greater, because the rock usually improves with depth. Therefore, given the importance that this index has on the results, the calculations have been done for a range of values from 60 to 75.

c) Uniaxial compression strength of the rock matrix (σ_c). Under the assumption that the rock will be saturated and assuming that the alteration degree of the rock mass is type W2 or W1, the corresponding value of σ_c varies between 35 to 45 MPa, respectively. They have been obtained after an extensive laboratory tests campaign.

The best estimation for a rock type W1 is 40 MPa. Since the minimum quality required will correspond to alteration degrees of W1-W2, it will also be calculated for 35 and 45 MPa.

d) Disturbance factor (D). According to Hoek, this parameter corresponds mainly to the human factor influence in the quality of the rock mass, once the desired level of the foundation is reached. However, the negative effect of a careless implementation of blasting is not expected to affect the 1 to 2 m shallower. In turn, stresses relaxation by excavating will be very small. Therefore, in these calculations, it has been assumed D = 0.

e) Specific weight of rock mass (γ). It has been assumed to be equal to 27 kN/m^3.

5.1.2 *Geometric parameters*

a) Foundation width (B). At the maximum high section the width is 21 m.
b) Foundation depth (H). The expected values of the rock mass are representative at a depth of 12 m.
c) Acting load inclination at the foundation (i_2) in relation to the vertical. According with the results obtained from the stress calculation, it has been fixed in 15º. Is a very determinant factor.
d) Natural ground inclination (α_1). The natural inclination of the river in the zone is practically null.
e) Inclination of the foundation (α_2). Transversally, the contact surface at the foundation level has an inverted slope of 10º. Its incorporation by simple form can be made decreasing the acting load inclination; finally, the adopted value for this angle is $i_2 = 5º$.
f) Acting load at downstream surface (σ_1). In this case a value of 0.324 MPa has been incorporated, acting vertically ($i_1 = 0º$).

5.2 *Results of the ultimate bearing capacity*

Taking into consideration that the maximum value of the vertical component of the load acting on the foundation is around 2,5 MPa, in Table 2 the results of the ultimate bearing capacity and the safety factors (SF) are shown for the analyzed ranges.

Table 2. Ultimate bearing capacity and safety factors for the methodology proposed.

	ULTIMATE BEARING CAPACITY q_h (MPa) and SF		
RMR	60	*65*	75
σ_c (MPa)	35	*40*	45
q_h without selfweight	68	*90*	145
q_h with selfweight	73	*96*	150
SF with selfweight	29	*38*	60

NOTE: In italics type the best estimation

5.3 *Summary*

After performed calculations, it can be asserted that:

- The ultimate bearing capacities that produce the rock mass failure affected by a double curvature arch dam foundation were calculated, using the plasticity theory and the Hoek & Brown (1980) failure criterion, following Serrano & Olalla (1994) methodology.
- It has been performed for the best estimation of the mean values of most of the parameters involved and for a reasonable and conservative range of the geomechanical index that represent the rock mass quality (RMR or GSI) and of σ_c.
- Depending on the adopted assumptions, the safety factors that have been obtained are around 40, and ranging from 30 to 60.

6 CONCLUSIONS

Different existing methods to determinate of the ultimate bearing capacity of rock masses and the corresponding safety factors, applied to a foundation of a double curvature arch dam on sound schists, have been evaluated. Most of these theories do not consider the main factors of the rock media that affect the foundation behavior. The results obtained show an extremely high variation.

The proposed methodology is based on the Serrano & Olalla formulations (1994). It applies the plasticity theory to the Hoek & Brown failure criterion (1980) and allows introducing the main parameters involved in the foundation behavior: acting load inclination, uniaxial compression strength, rock quality, depth of the foundation level, foundation width, etc.

The used method take into consideration several parameters, both geotechnical (m_i, RMR or GSI, σ_c, γ) and geometric (B, H, i_2, α_1, α_2).

The maximum load that will act on foundation was previously calculated using a finite element program that models the dam and its foundation; it provides a value around 2.5 MPa, acting with a maximum inclination in relation to the vertical of 15º.

The application of this method for this 100 m high double curvature arch dam, a safety factor around 40 was obtained, much greater than the required coefficient of 3 suggested in most of analyzed codes and regulations (although the obtained factors vary between 14 to 26, with exception of GCOC 2009 and ROM 2005 that are 3.5). Serrano & Olalla (1996), based on several statistical hypothesis, propose a minimum safety factor of about 20 to 25, for an allowable failure probability less than 10^{-4}, for a strength and quality of the rock mass similar to this real case.

Based in all these arguments, it is possible to state that the performed calculations allow confirming the goodness of the taken decision about the depth of the foundation, with the expected quality for the rock mass and its corresponding geomechanical properties.

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